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Which Dark Matter Halo properties effect the size of the resulting galaxy?

Controlled Experiments on Dark-Matter Halo Structure and Galaxy Morphology

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In China!



This is a preprint of the article, meaning it is not published yet!

In a nutshell:

The authors use isolated galaxy simulations to find out how different halo sizes, spin, concentration, and baryon fraction regulate galaxy size. They find that halo mass is the most important factor, but halo spin and concentration also play a role. They use a parameter grid to test how different halo properties affect the size of the resulting galaxy. The resulting half-mass radii of stars and cold baryons depend systematically on halo structure and baryon content. They find that halo spin is a weak predictor of size, but halo concentration is a strong predictor. They also find that baryon fraction has a subtle, non-monotonic impact on size, by modulating how galaxy size depends on halo spin. Our results clarify which secondary parameters of host dark-matter haloes regulate the scatter in galaxy sizes at the mass–size dwarf-galaxy scale.

Key words: galaxies: formation – galaxies: haloes – galaxies: dwarf – galaxies: structure



Background:

In the standard Λ CDM paradigm of structure formation, dark matter (DM) haloes are the primary building blocks of galaxies. They provide structures, haloes, that provide gravitational potential wells in which galaxies form (e.g. White 1983; Springel et al. 1984). Theoretical models and numerical simulations based on this paradigm have shown that the properties of DM haloes, such as their mass function, spatial clustering, and internal structure (e.g. Press & Schechter 1975; Mo et al. 1997; Springel et al. 2005). Given these mature theories of DM haloes, a crucial step towards a theoretical framework of galaxy formation lies in establishing the galaxy–halo connection — the set of rules that govern how galaxies populate and evolve within these haloes (see e.g., Wechsler & Tinker 2016, for a review).

A central question in the galaxy–halo connection is how the properties of dark-matter haloes regulate the morphology of the resulting galaxies, particularly their characteristic sizes. Halo mass has been widely recognized as the primary driver of galaxy morphology, a conclusion supported by several complementary lines of evidence. On average, galaxy size exhibits an approximately linear scaling with the virial radius of the host halo (e.g. Kravtsov 2013; Somerville et al. 2018), $R_{\text{gal}} \propto 0.02 R_{\text{vir}}$. At a given epoch, the virial radius R_{vir} is determined by the total mass of the host halo, which can be inferred from stellar masses using stellar mass–halo mass relations derived from abundance matching (e.g. Moster et al. 2013; Behroozi et al. 2013).

Higher Halo Mass = Larger galaxy size

Second, cosmological hydrodynamical simulations show that disc morphology is more diverse in galaxies above the characteristic mass threshold, $M_{\text{vir}} \gtrsim 10^{11.5} M_{\odot}$ (Dekel et al. 2020; Lapiner et al. 2020). Jiang et al. (2022) find that galaxies with $M_{\text{vir}} \sim 10^{11.5} M_{\odot}$ display striking morphological diversity, with stellar surface densities spanning more than two orders of magnitude, and ultra-diffuse galaxies coexisting alongside compact dwarfs (see e.g. Sales et al. 2022). Thus, while halo mass sets the overall scale of galaxy size and morphology, the substantial scatter around these relations indicates that additional halo properties beyond mass may play an important role in regulating the final extent of galaxies.

The origin of this scatter remains debated. The angular momentum of the dark-matter halo, often parameterized by the dimensionless spin parameter λ (Peebles 1969; Bullock et al. 2001), has been considered a primary candidate for regulating galaxy size. Models of galaxy formation, assuming conservation of specific angular momentum during baryonic infall, predict that galaxy size scales with halo spin and virial radius, $R_{\text{gal}} \propto \lambda R_{\text{vir}}$ (e.g., Fall & Efstathiou 1980; Mo et al. 1997).

However, hydrodynamical cosmological simulations reveal surprisingly mixed views and somewhat weak correlation between the spin of a galaxy and that of its host dark-matter halo (Jiang et al. 2019; Rohr et al. 2022). In particular, using two suites of zoom-in simulations, Jiang et al. (2019) find that the galaxy half-stellar-mass radius correlates more tightly with halo concentration than virial radius, following an empirical relation $r_{1/2,*} \propto c^{-1} R_{\text{vir}}$. Focusing on central dwarf galaxies with stellar masses $M_{*} \sim 10^{7-9} M_{\odot}$ in the FIREbox simulation, Rohr et al. (2022) show that the scatter in

Even literature gives contradicting results

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2 Even different simulations give different results!

the halo size, baryon fraction, inner density profile, and halo virial radius exhibits little correlation with halo spin (or concentration), suggesting that baryonic processes overwhelm halo properties in setting the sizes of these bright dwarf systems. The strength of these correlations varies across simulation suites. For example, Liang et al. (2025a,b) find that in the IllustrisTNG-50 simulations, galaxy size at fixed halo mass exhibits a modest positive correlation with the halo spin parameter as well as a slight anti-correlation with halo concentration. Taken together, these studies indicate that different halo properties and baryonic physics contribute with varying importance, and a unified understanding of the primary drivers of galaxy size at fixed halo mass remains elusive.

In addition to the mixed results from cosmological simulations, many of these studies share a conceptual limitation: the relations between galaxy size and host-halo structure are typically established using instantaneous halo properties, whereas galaxy sizes should, in principle, be determined by the evolution of their progenitor haloes at the time when the galaxy formed. Using cosmological simulations to connect galaxy properties to progenitor halo structure, however, presents two major challenges. First, there is no consensus on how to define the halo “formation” time. Common definitions include the epoch when a halo first assembles half of its final mass (e.g. Lacey & Cole 1993; Smith & Tormen 2004), the end of the last accretion phase (e.g. Zhao et al. 2007), or the time of the last major merger (e.g. Li et al. 2008). It remains unclear which of these definitions is most appropriate for the study of galaxy formation epoch for establishing galaxy–halo connections. Second, even if a specific time is chosen, it is often difficult to extract the evolution of halo structural parameters requires reprocessing particle-level data across the entire simulation, which is computationally expensive and intensive.

Another limitation of previous studies is that cosmological simulations is the restricted range of halo properties that they typically probe. For example, most simulations designed to study dwarf galaxy structure employ zoom-in techniques of small simulation volumes (e.g. Wang et al. 2015; Hopkins et al. 2018). While these approaches achieve the numerical resolution required to resolve dwarf galaxies, they inevitably sacrifice statistical power and limit the diversity of halo structural properties that they can probe. This presents a significant challenge for addressing questions such as how the inner density profile of a host halo may influence the formation of galaxies. In the FIRE-2 and FIRE-3 simulations, haloes with masses around $10^{11} M_{\odot}$ almost uniformly show that inner density profiles (e.g. Lindlich et al. 2020; Lazar et al. 2020), whereas haloes in the IllustrisTNG simulations remain predominantly cusp profiles across a wide range of halo masses (Bose et al. 2019).

These considerations motivate complementary cosmological analyses with controlled numerical experiments designed to isolate specific physical dependencies. In controlled simulations, there is no ambiguity in the definition of formation time, as the properties of the system at formation are fully specified by the initial conditions. Moreover, halo parameters can be varied systematically, enabling a wide baseline of halo structural properties by construction. In this series of studies, we adopt idealized (isolated) galaxy simulations to identify which secondary dark-matter halo properties, such as halo mass, host halo inner density profile, and halo spin, are most important in setting galaxy size. In this first paper of the series, we focus on a single halo mass, $10^{11} M_{\odot}$, and explore how variations in halo properties affect the size of bright dwarf galaxies and the mass scale at which structural diversity is maximized (e.g. Di Cintio et al. 2014; Oman et al. 2015; Lazar et al. 2020). We systematically explore how variations in halo spin, concentration, inner density slope, and baryon fraction modu-

late galaxy size at fixed halo mass. In subsequent papers in this series, we plan to extend the study to a broader range of halo masses and examine the halo conditions governing additional aspects of galaxy morphology and size.

This paper is organized as follows. Section 2 describes the design of our controlled numerical experiments, the implementation of the hydrodynamical simulations, and the analysis methods. Section 3 presents the correlations between galaxy size and initial halo properties and establishes a rank ordering of the relative importance of these halo parameters in regulating galaxy size. In Section 4, we discuss the implications of our findings for galaxy size predictors commonly employed in semi-analytic and empirical models. We summarize our conclusions in Section 5. Throughout this work, we define haloes as spherical overdensities of 200 times the critical density of the Universe at $z = 0$, with $\rho_{\text{crit}} = 126.08 M_{\odot} \text{ kpc}^{-3}$, and denote halo virial radius and mass as R_{vir} and M_{vir} , respectively.

How they did it:

In this section, we present the details of our controlled simulation suite. We describe and motivate the choices of halo parameters and halo mass. Galaxy size and structural properties are defined in our models (Section 2.1), summarize the numerical code GIZMO and the FIRE-3 galaxy formation prescriptions (Section 2.2), and outline the analysis tools used in this work (Section 2.3).

2.1 Halo parameters and simulation setups

Here we focus on the impact of halo structural parameters on galaxy size and shape. The halo mass is fixed at $10^{11} M_{\odot}$, a choice of this mass scale for several reasons. First, structural diversity of galaxies is maximized at this halo mass (e.g. Di Cintio et al. 2014; Oman et al. 2015; Lazar et al. 2024). Second, characterizing galaxy sizes is most difficult at low masses, where morphological structures of the stable disc galaxies begin to emerge at approximately this halo mass (e.g. Di Cintio et al. 2014; Lazar et al. 2020). Third, although more massive haloes ($\sim 10^{12} M_{\odot}$) can also host stable discs, the influence of accreting supermassive black holes is, in principle, non-negligible at those masses, while a proper treatment of such effects is not yet fully captured by current subgrid prescriptions.

That said, we have verified with a set of test simulations spanning different halo masses that our models approximately reproduce key scaling relations, such as the stellar mass–halo mass relation, and exhibit the strongest cusp–core transitions in the halo mass regime characteristic of—and consistent with—those found in cosmological FIRE-2 simulations (Lazar et al. 2020).

The parameters varied in our simulations are defined as follows.

Halo concentration is defined as $c_2 = R_{\text{vir}}/r_c$, where r_c is the radius at which the logarithmic density slope equals -2 . DM haloes are initialized and fitted using the Dekel-Zhao (DZ) profile (Fritz et al. 2008), which includes an additional degree of freedom relative to the Navarro et al. (1997, NFW) profile and allows for a shallower inner density slope at small radii. The DZ profile is given by:

$$\rho(r) = \frac{\rho_0}{x^a (1 + x^{1/2})^{2(3.5-a)}}, \quad (1)$$

where $x = r/r_c$, with r_c a scale radius defining the concentration $c = R_{\text{vir}}/r_c$. The parameter a specifies the asymptotic inner density slope as $r \rightarrow 0$, and ρ_0 is characteristic density related to the virial

Table 1. Values adopted in the controlled simulations suite for each parameter in the factorial grid of c_2 , s_1 , λ , and f_b , at a fixed halo mass of $M_{\text{vir}} = 10^{11} M_\odot$ (Section 2.1). The values are the actual values they use for each parameter:

Properties	Values
c_2	3, 10, 20
s_1	0.0, 0.5, 1.0, 1.5
λ	0.02, 0.04, 0.06, 0.08
f_b	0.03, 0.07, 0.14

overdensity Δ via:

$$\rho_0 = (1 - a/3)\bar{\rho}_0, \quad \bar{\rho}_0 = c^3 \mu \Delta \rho_{\text{crit}}, \quad (2)$$

with

$$\mu = c^{a-3} (1 + c^{1/2})^{2(3-a)}. \quad (3)$$

The DZ parameters c and a are not always the most convenient to vary. The inner density slope, s_1 , differs from the commonly used concentration parameter c_2 , which coincides with the halo concentration c_2 in the DZ formalism. The two are related by

$$s_1 = \frac{R_{\text{vir}}}{r_1} \left(\frac{1.5}{2 - a} \right)^2, \quad (4)$$

- The density slope, s_1 , is defined as the logarithmic slope $-d \ln \rho / d \ln r$ evaluated at 1% of the virial radius. This quantity can be expressed in terms of the DZ parameters c and a as:

$$s_1 = \frac{a + 3.5c^{1/2}(r_1/R_{\text{vir}})^{1/2}}{1 + c^{1/2}(r_1/R_{\text{vir}})^{1/2}} = \frac{a + 0.35c^{1/2}}{1 + 0.1c^{1/2}}, \quad (5)$$

where $r_1 = 0.01 R_{\text{vir}}$.

- Halo spin – The angular momentum of the halo

$$\lambda = \frac{j_{\text{vir}}}{\sqrt{2} R_{\text{vir}} V_{\text{vir}}}, \quad (6)$$

where $j_{\text{vir}} = J_{\text{vir}}/M_{\text{vir}}$ is the specific angular momentum of the halo, and V_{vir} the circular velocity evaluated at the virial radius R_{vir} .

- Baryonic fraction – Amount of normal matter in comparison to the total amount (normal + dark matter) f_b
- are defined as the ratio of the total baryonic mass to the virial mass. Here, we emphasize that f_b is not necessarily equal to the gas-to-halo baryonic fraction of ≈ 0.165 . Observations of nearby galaxies reveal substantial scatter in baryon fraction across a wide range of halo masses (e.g. Mancera Piña et al. 2023). Consistent with this, cosmological hydrodynamical simulations such as FIRE-2 also show that f_b deviates from the cosmic fraction at halo masses below the Milky Way and exhibits significant scatter (Hafen et al. 2019). We therefore include f_b as a parameter in our grid and explore how variations in f_b affect the resulting galaxy properties. We adopt a fiducial value of 0.07, motivated by the results of Hafen et al.

We construct a nearly factorial parameter grid that samples the full range of properties (c_2, s_1, λ, f_b) at the listed values (Table 1). These parameter choices are designed to sample the range of values commonly found in cosmological simulations. For example, halo concentration exhibits a lower bound of ~ 3 , a median value of ~ 10 , and a $\sim 2\sigma$ upper-tail value of ~ 20 at the halo mass of interest (e.g. Bullock et al. 2000). Similarly, the halo spin parameter has a median value of ≈ 0.04 , with 0.02 and 0.08 corresponding to the approximate 2σ tails of the distribution (Bullock et al. 2001). The inner density slope values span the full range relevant to the cusp–core problem (e.g. Bose et al. 2019; Freundlich et al.

2020), from completely flat cores ($s_1 = 0$) to steep cusps ($s_1 = 1.5$). In principle, this grid would yield $3 \times 4 \times 4 \times 3 = 144$ parameter combinations. However, we do not include flat-top profiles, which are not physically meaningful solutions for the combination $(c_2, s_1) = (20, 0)$, and we therefore exclude all models with this pairing. The resulting design consists of $N = 132$ distinct simulation runs, which form the main sample analyzed in this work.

For each parameter combination, we construct an initial condition (IC) consisting of a DZ DM halo and a gaseous component in approximate hydrostatic equilibrium. There is no unique prescription for establishing exact equilibrium between the gas and the host halo; instead, we follow the approach of Kim & Lee (2013), in which the gas distribution that is initialized to be self-similar to the DM halo and is then allowed to relax toward equilibrium under gravity and hydrodynamics alone.

If you want to know how the particles are initialized in the simulation and the assumptions you put on the gas, metallicity, temperatures etc. you can look here.

The initial temperature of the gaseous component is set to the halo virial temperature,

$$T_{\text{vir}} = \frac{\mu m_p}{2k_B} \frac{GM}{R_{\text{vir}}}, \quad (7)$$

We adopt a metallicity of $Z = 0.003 Z_\odot$, following Kim & Lee (2013).

To set the initial specific angular momentum, we follow the empirical relation of Kim et al. (2011),

$$j(< M) = \dots \quad (8)$$

Where $j(< M)$ is the specific angular momentum enclosed within mass M . We determine the target specific angular momentum by requiring that the integrated angular momentum of the gas and stars produces a target spin parameter¹. Given a target specific angular momentum to each component, we assign particle velocities accordingly.

We then evolve this configuration for 100 Myr under gravity and hydrodynamics only, allowing the system to relax. During this relaxation period, the gas and stars settle into a steady state. The halo parameters listed in our experimental design (Table 1) therefore correspond to the values measured after this relaxation period. The relaxed systems are subsequently evolved for an additional 3 Gyr using the GIZMO code with full physics enabled. This duration is long enough for the gas and stars to reach a steady state and is sufficient for the galaxies to reach a quasi-steady state. We analyze the galaxy properties at the end of the full-physics simulations and quote the halo properties at the conclusion of the relaxation phase.

Once galaxies reach a steady state then the galaxy sizes are measured.

¹ This trial target value is different from (slightly higher than) the intended grid value listed in Table 1, because the full-physics simulations are initialized after the system has relaxed, and all quoted values correspond to measurements taken after this relaxation phase.



Circles used to compare how much mass or radius is within a certain region.

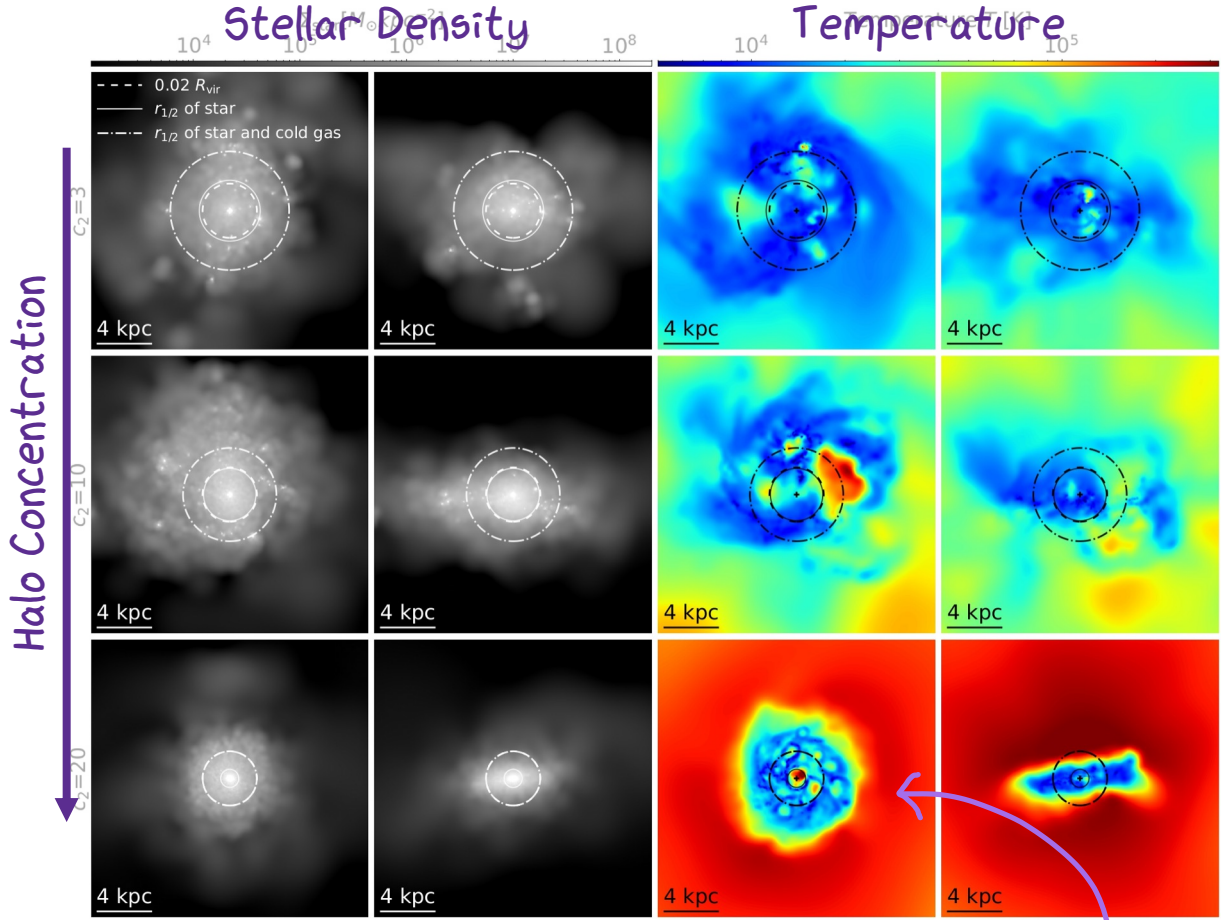


Figure 2. Same as Fig. 1, but varying halo concentration c_2 from 3 to 20 while keeping all other parameters at their fiducial values: spin $\lambda = 0.0$, inner DM density slope $s_1 = 1$, and baryon fraction $f_b = 0.07$.

Galaxy sizes remain the same with inner density slope unless towards the edge of the parameter range where it greatly reduces.

Galaxy sizes remain the same with inner density slope unless towards the edge of the parameter range where it greatly reduces. Varying the baryon fraction f_b has a significant impact on galaxy morphology and size. As shown in Fig. 4, for $f_b \lesssim 0.07$, galaxy sizes are similar, with stellar half-mass radii $r_{1/2,*} \sim 0.02R_{\text{vir}}$ and cold-baryon half-mass radii $r_{1/2,*+cg}$ approximately twice as large. In this low- f_b regime, the cold gas forms a relatively settled disc, as readily indicated by the temperature maps. In contrast, for $f_b \sim 0.14$, which is close to the cosmic baryon fraction, the system undergoes centrally concentrated star formation and develops a compact stellar component. Even the cold gas is no longer as diffuse in a well-ordered, disc-like configuration. These results imply that the baryon fraction where star formation can occur which shapes morphology and size.

Galaxy size decreases as gas falls deeper into the potential well!

It strongly regulates where gas can reach star-formation conditions, thereby shaping both galaxy morphology and size – an effect that becomes particularly pronounced at high f_b .

We caution that the f_b dependence may be exaggerated in our idealized setup relative to fully cosmological simulations. In our high- f_b model, an amount of gas that is close to the cosmic baryon fraction is initialized instantaneously within the moderately massive halo, rather than being bunched gradually over cosmic times in a gradually growing halo. As a result, dense central gas may instantly trigger star formation, leading to a central starburst. Despite this caveat, our simplified experiments offer valuable physical insights. They suggest that the onset of star formation in a moderately massive halo (e.g., $M_{\text{vir}} \sim 10^{11} M_\odot$) likely leads to the formation of compact galaxies, whereas stellar feedback is included through the FUSE-3 physics. Second, baryon fractions below the cosmic mean are a natural requirement for bright dwarf systems to develop extended sizes and disc morphologies.

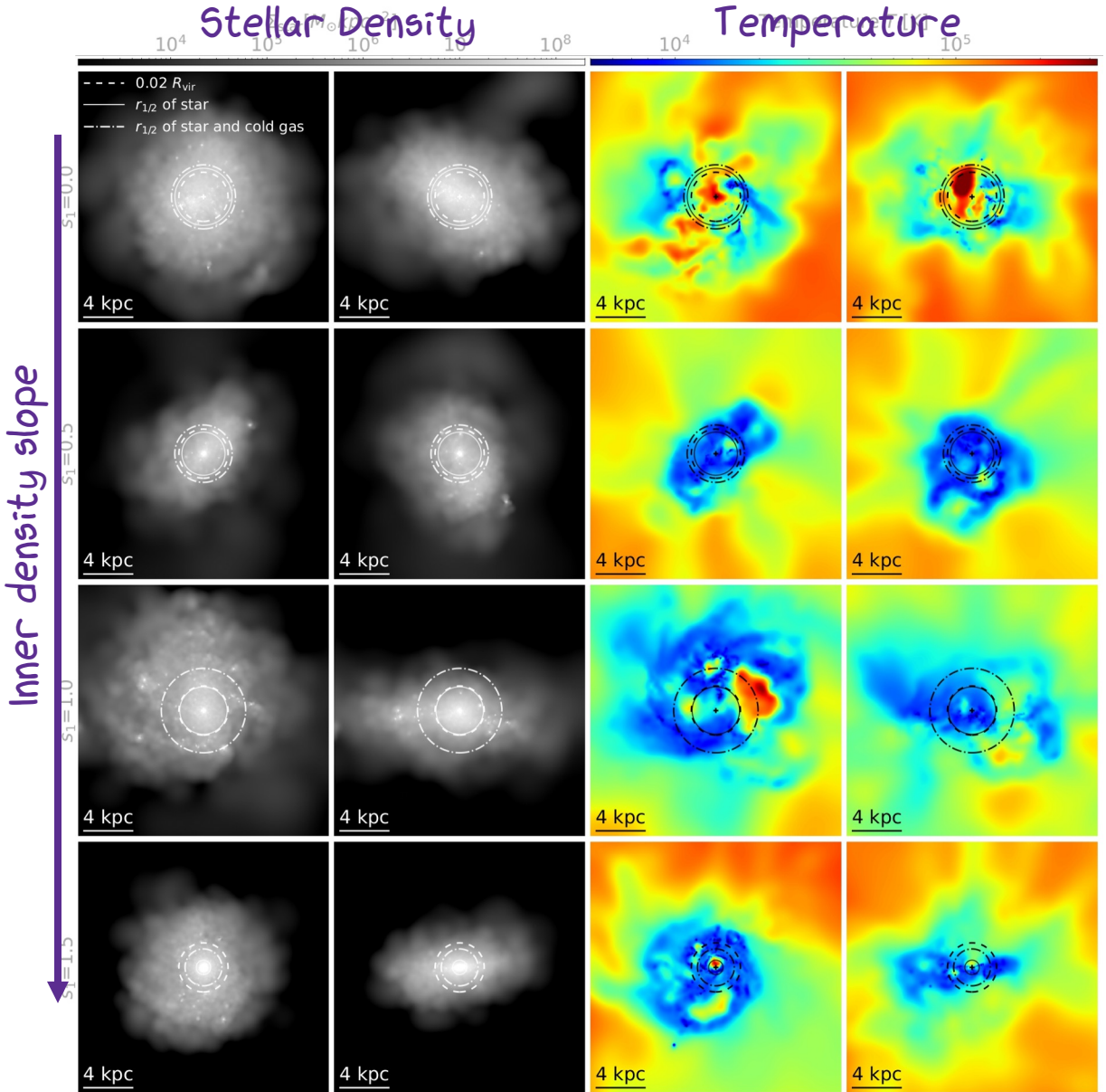
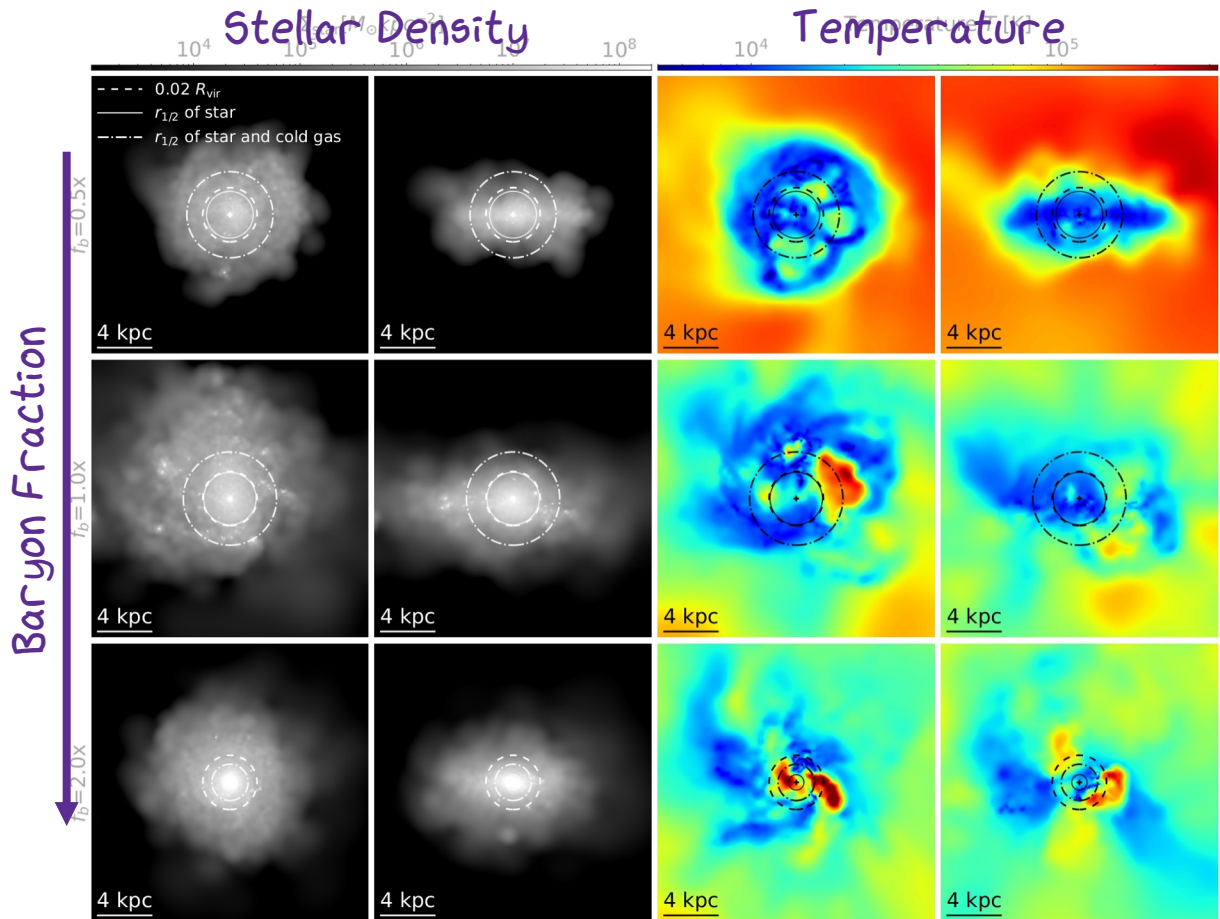


Figure 3. Same as Fig. 1, but varying the inner logarithmic slope of dark matter density s_1 from 0 to 1.5, while keeping all other parameters at their fiducial values: spin $\lambda = 0.04$, concentration $c_2 = 10$, and baryon fraction $f_b = 0.07$.

Mostly the same until the bottom row, where the outer most ring shrinks dramatically!
 This outer ring corresponds to the radius that contains half the mass of the gas and stars.



Some reduction in size here!

Figure 4. Same as Fig. 1, but varying the baryon fraction f_b relative to the fiducial value of 0.07 (Hafen et al. 2019) from 0.5 $\times$ to 2.0 $\times$, while keeping all other parameters at their fiducial values: spin $\lambda = 0.04$, concentration $c_2 = 10$, and logarithmic dark-matter-density slope $s_1 = 1$.

Back to quantifying the effects!

The visual examinations of the density maps and temperature maps in Section 3.1 confirm that all the four parameters impact galaxy morphology and size. It is natural to aim for a quantitative result on the relative importance. Since there is no simple concordance metric for relative importance, we adopt complementary measurements summarized in Table 2.

In this section, we consider two importance proxies. The first is the Pearson correlation coefficient $\mathcal{R}$ between each halo parameter and galaxy size, $r_{1/2,*}$, $r_{1/2,*}$ and $r_{1/2,*}$ indicate the significance of the linear relationship in the marginal sense. The second class of proxies is derived from the full factorial response surface model (RSM) in Section 3.1. For a given factor, we assess its contribution by comparing the full RSM to a reduced model that excludes the factor. The difference in fit defines a partial R^2 (coefficient of partial determination), namely the fraction of the residual variance

explained by the removed terms. We report both a *main-effect* partial R^2 , obtained by removing the linear and quadratic terms of that factor, and a *total-effect* partial R^2 , which additionally removes all interaction terms involving the same factor (see Appendix A for details).

They also use that response model mentioned earlier and compare it against the model without that parameter included at all to see the effects. R^2

Table 2. Summary of complementary importance proxies used to quantify the dependence of the stellar half-mass radius, $r_{1/2,*}$, on the four halo parameters. Pearson’s $\mathcal{R}$ measures the linear correlation between each parameter and $r_{1/2,*}$ across the full simulation sample, with the sign indicating the direction of the trend. The quadratic response-surface model (RSM) provides two conditional importance scores: the effect and total effect, obtained from a joint (linear + quadratic) coefficient-block test for each parameter, and a total-effect partial R^2 obtained from a nested block test that additionally includes all pairwise interaction terms involving that parameter (definitions and HC3-robust inference are described in Appendix A). Random-forest (RF) entries report normalized feature importance (each RF row sums to unity across the four parameters); we list four RF estimators—MDI, out-of-fold permutation (OOB), permutation importance (PI), and SHapley (SHAP).

importance proxy	c_2	λ	s_1	f_b
Pearson $\mathcal{R}$	−0.49	+0.35	−0.44	+0.20
RSM main-effect partial R^2	0.463	0.477	0.492	0.033
RSM total-effect partial R^2	0.579	0.523	0.544	0.210
RF MDI	0.303	0.241	0.267	0.188
RF permutation (OOF)	0.389	0.231	0.200	0.180
RF permutation (OOB)	0.395	0.235	0.193	0.178
RF SHAP	0.372	0.245	0.255	0.128

The panel of Fig. 5, consistent with a trend increase in galaxy size with f_b at $f_b \lesssim 7$, followed by a decrease at larger f_b , a trend that is particularly pronounced for some cases with $\lambda = 3$ and 10. We have verified that the non-monotonic behavior of the f_b trend is also present at moderate halo spin values ($\lambda \approx 0.04$ –0.06). Only at the highest spin values ($\lambda \gtrsim 0.1$) does the trend reverse, with f_b whereas at the lowest spin values galaxy size decreases monotonically with

This behavior is illustrated more clearly in Fig. 6, which visualizes the joint effect of varying pairs of halo parameters at a time. The parameter spaces spanned by the four halo parameters c_2 , s_1 , and λ exhibit nearly linear gradients, indicating that the effects of individual parameters can, to good approximation, be treated as additive. A notable exception arises in the parameter spaces involving the baryonic fraction f_b , where the contours display pronounced curvature. This behavior can be interpreted in a complementary way. First, as noted above, defined c_2 , s_1 , and λ , the dependence of galaxy size on f_b is generally non-monotonic. Second, the influence of halo concentration and spin on galaxy size becomes significantly stronger at higher baryonic fractions. Third, as discussed in §3.2, the baryonic fraction modulates how sensitively galaxy size responds to halo structure, rather than acting as a uniform regulator of size.

3.5 Feature importance ranking from random-forest regression as a sensitivity controlling parameter.

As an alternative to the importance scores from the response-surface model, we train a random-forest regressor (Breiman 2001) to predict the stellar half-mass radius $r_{1/2,*}$ from the four halo parameters (c_2 , s_1 , λ , f_b) with sklearn (Pedregosa et al. 2011). Here our goal is not an optimal predictive model, but an importance ranking that is insensitive to the quadratic functional assumption in Section 3.2. After experimenting with multiple importance estimators (Appendix B), we find that the permutation importance estimator c_2 is consistently the most informative predictor of galaxy size, with a relative importance score of ~ 0.2 –0.4 depending on the specific metric, while halo spin λ and inner slope s_1 have comparable contributions of ~ 0.2 –0.25, followed by the baryonic fraction f_b , which scores ~ 0.15 . Table 2 summarizes the importance metrics and scores discussed

Rank of parameter importance:

throughout this section. Halo concentration stands out as the primary size regulator followed by the spin parameter.

1. Concentration

2. Spin

3. Slope

Though some of their metrics contradict!

Let’s Discuss:

4.1.1 Comparison with the Mo98 model

The classical disc-formation model of Mo et al. (1998) (Mo98) predicted the galactic size by relating the stellar half-mass radius and spin of the host halo, under the assumption that the baryons forming the disc are able to retain a fraction of the angular momentum as the halo and retain a fraction of f_d during galaxy formation. In this framework, the disc half-mass radius is given by

$$r_{1/2,*}^{(\text{Mo98})} = \frac{1.678}{\sqrt{f_d}} f_c \lambda R_{\text{vir}} f_b^{1/2} f_{\text{ret}} \quad (10)$$

where the size scales linearly with the angular-momentum retention factor f_{ret} .

the sizes are governed by those parameters.

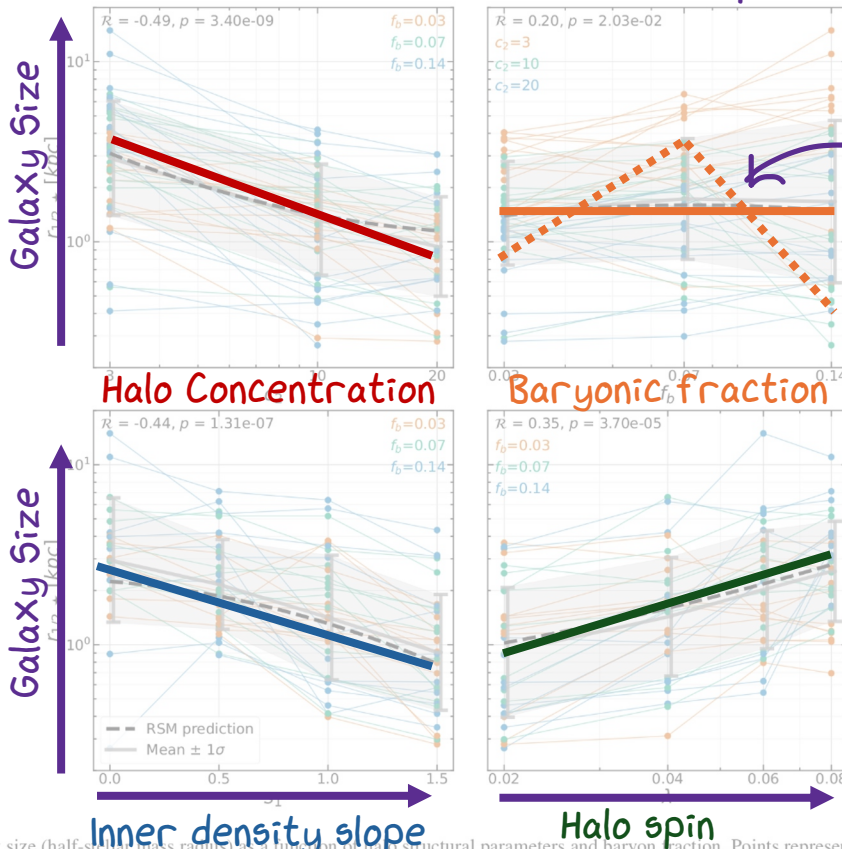
Here $m_d = M_*/M_{\text{vir}}$ and $j_d = J_d/J_{\text{vir}}$ denote the mass and angular-momentum fractions of the galaxy relative to its host halo, respectively. The factor f_c characterizes the energy structure of the halo and is defined as the ratio of the halo’s binding energy to that of a singular isothermal sphere of the same mass. The factor f_R accounts for the contraction of the halo in response to the gravitational potential of the disc.

The original Mo98 formula assumes that haloes follow an NFW profile. Here, our simulations adopt the DZ profile in order to allow explicit control over the inner density slope. To enable a fair comparison, we therefore adapt the Mo98 model to DZ haloes and compute a predicted stellar half-mass radius $r_{1/2,*}^{(\text{Mo98})}$ for each simulation. In practice, this entails computing the factors f_c and f_R factors using the DZ profile, and measuring j_d and m_d directly from the simulated galaxies, strictly following their original definitions based on the stellar mass and stellar angular momentum. Details of our implementation of the Mo98 model for DZ haloes, as well as validation tests, are provided in Appendix C.

The left panel of Fig. 7 compares the Mo98-predicted sizes, $r_{1/2,*}^{(\text{Mo98})}$, with the stellar half-mass radii measured in the simulations, $r_{1/2,*}$. The model predictions exhibit a clear systematic offset relative to the simulation results, in that the Mo98 model typically under-predicts galaxy sizes by a factor of ~ 5 . Aside from this systematic offset, however, the model reproduces an approximately linear scaling with the simulated galaxy sizes.

Colour-coding individual galaxies by the angular-momentum retention factor f_{ret} reveals that the simulated typical f_{ret} values well below unity, spanning a wide range from ~ 0.01 to $\lesssim 1$, with a typical value of ~ 0.2 . This indicates that the Mo98 model overestimates the Mo98 size predictions after artificially boosting the retention factor f_{ret} to a fixed value of unity (the open triangles in Fig. 7). This ad hoc correction brings the predicted sizes into close agreement with the one-to-one relation. The failure of the simple angular-momentum retention assumption is expected if star-forming gas is preferentially

10 *G. Sengupta et al.* Here is the plots for the Pearson statistic which tests for a linear relationship:



This doesn't always capture the underlying behavior

Figure 5. Galaxy size (half-stellar mass radius) as a function of halo structural parameters and baryon fraction. Points represent individual simulation results, coloured by a secondary parameter (f_b or c_2 , as indicated). Thin, colour-matched lines connect simulations in which only the horizontal-axis quantity is varied while all other parameters are fixed. Connected grey symbols show the mean stellar half-mass radius, $r_{1/2,*}$, with error bars and the light-grey bands indicating the 1σ scatter. The symbol positions are slightly offset horizontally to improve visual clarity. The black dashed curve shows the mean prediction of the best-fit quadratic response-surface model (RSM), obtained by varying the parameter of interest while fixing all others at their midpoint values. Pearson correlation coefficients, R , and the corresponding p -values are quoted.

drawn from the low-angular-momentum tail of the gas distribution and if stellar feedback redistributes angular momentum.

4.1.2 Comparison with simple empirical size predictors

Jiang et al. (2019) (J19) found that galaxy spin (and hence size) is only weakly correlated with halo spin in their cosmological zoom-in simulations. They proposed an empirical, concentration-based size predictor of the form

$$r_{1/2,*} \approx 0.02(1+z)^{-0.2}(c_*/10)^{-0.7}R_{\text{vir}}. \quad (12)$$

In this relation, the dependence on halo concentration was introduced in part to account for the strong redshift-dependence of the ratio between galaxy size and halo virial radius seen in their cosmological simulations – because the concentration itself evolves with mass and redshift and naturally introduces an implicit redshift dependence. J19 noted that the Mo98 formalism also contains a concentration dependence via the factor f_c . But this dependence is much weaker. Moreover, the redshift dependence of the concentration alone cannot fully explain the redshift dependence of $r_{1/2,*}/R_{\text{vir}}$ seen in the simulations, motivating the introduction of the additional redshift scaling

factor $(1+z)^{-0.2}$. More recently, analyses of large-box cosmological simulations have shown that, even at fixed mass and redshift, disc galaxies in the same mass range with different halo spin (Jiang et al. 2025a,b). This raises the question of whether the concentration dependence of the size predictor proposed by J19 reflects an imprint of the halo structure itself.

The middle panel of Fig. 7 compares our measured stellar half-mass radii to the J19 predictions. On average, our idealized isolated simulations reproduce the same qualitative concentration trend reported by J19, suggesting that the dependence of galaxy size on concentration does not require a cosmological environment, but instead emerges from internal dynamical processes that govern galaxy formation within virialized haloes.

The right panel of Fig. 7 compares our measured half-mass radii

⁴ Because the J19 relation depends explicitly on halo concentration and our simulations suite samples three discrete values of concentration, the predicted sizes populate three narrow bands. A small amount of scatter is present because the target values of $c_2 = 3, 10$, and 20 are those measured after the initial relaxation stage. These values were achieved through trial and error and cannot be matched exactly to the intended targets.

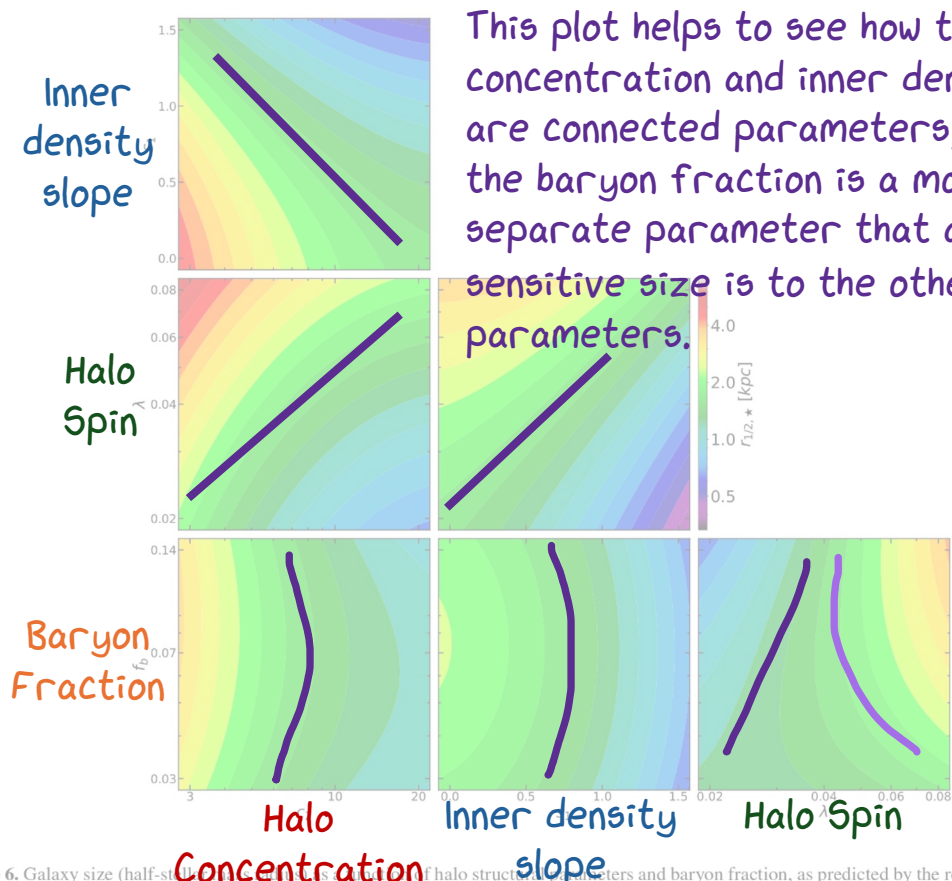


Figure 6. Galaxy size (half-stellar radius) as a function of halo structural parameters and baryon fraction, as predicted by the response-surface model fitted to our simulation results. In each panel spanning a pair of parameters, the remaining two parameters are held fixed at their midpoint values. The halo structural parameters exhibit largely additive behavior, reflected by quasi-parallel contours. In contrast, panels involving the baryon fraction show non-separable responses, with the f_b - λ plane displaying the strongest covariance. This indicates that the baryon fraction modulates the sensitivity of galaxy size to variations in halo structure.

to a simple predictor based on halo spin (Somerville et al. 2018). Since there is so much scatter in the plots on the next page, the authors conclude that considering all the parameters together is pretty important.

4.2 The impact of the galaxy definition

A practical but often under-emphasized issue in size modeling is that size definitions are not unique. Analytic disc models such as Mo98 are formulated for baryonic discs (stars plus cold gas), whereas many simulation and observational comparisons focus on stellar-only size measurements. Cold or neutral gas discs are usually thinner and more extended than their stellar counterparts: nearby H I surveys find that the ratio of H I to optical size is typically $\gtrsim 1$ –2 (e.g. Broeils & Rhee 1997). Simulations likewise produce neutral-gas discs that extended beyond the stellar body (e.g. Marinacci et al. 2017).

Fig. 8 quantifies the impact of defining the galaxy using stars alone versus stars plus cold gas. The half-mass radius of the cold baryonic component is systematically larger than the stellar-only radius, although the offset is modest compared to the full scatter of the sample. This definition-dependent difference affects the performance of the Mo98 model, in which the predicted disc scale depends explicitly on the angular-momentum retention factor, $f_j = j_d/m_d$.

The point of Fig. 8 is not the under-stellar-only definition, most systems lie well below the $j_d = m_d$ relation. Which cold gas is included, however, makes a difference to the value of m_d , indicating more efficient angular-momentum retention. This behavior is consistent with the expectation that a larger fraction of a disc galaxy’s angular momentum resides in the cold gaseous component (e.g. Mo98). As a result, the radii predicted by the Mo98 model increase, and the agreement with the measured sizes improves modestly when the galaxy is defined to include both stars and cold gas.

Key Takeaways:

In this paper, we have used a suite of controlled, idealized galaxy simulations to isolate how dark-matter halo structural param-

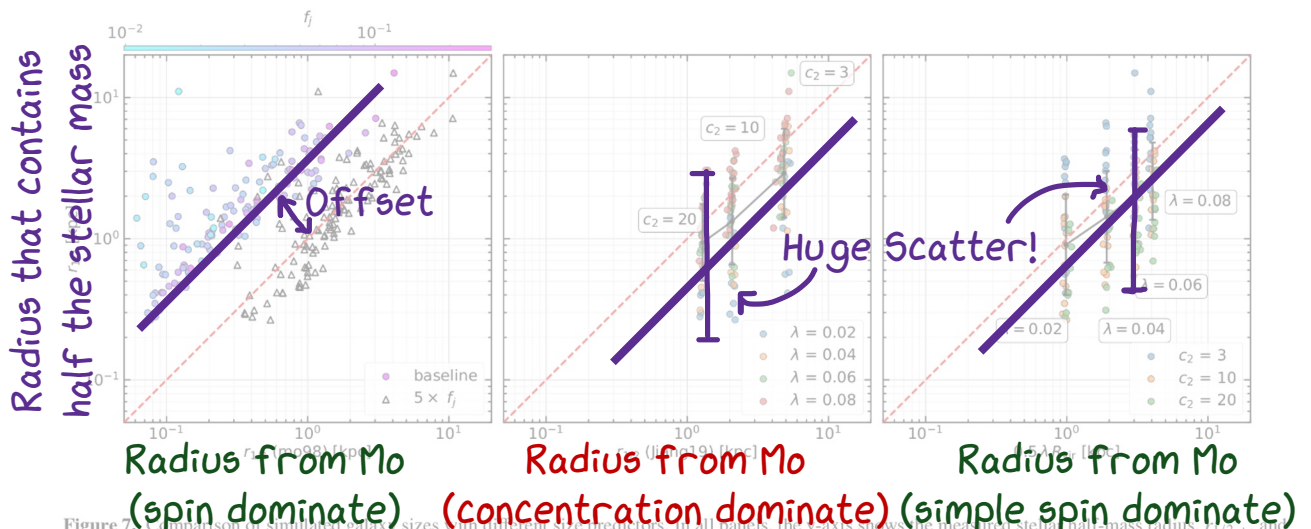


Figure 7. Comparison of simulated galaxy sizes with different size predictors. In all panels, the y-axis shows the measured stellar half-mass radius, $r_{1/2,*}$, and the red dashed line marks equality. *Left panel:* the Mo et al. (1998) (Mo98) predictor. The x-axis shows the predicted stellar half-mass radius $r_{1/2,*}^{(Mo98)}$. Filled circles show the predictions and are coloured by the angular-momentum retention factor $f_j \equiv j_d/m_d$. Open triangles illustrate the effect of boosting f_j by a factor of 5, which largely removes the systematic underprediction. *Middle panel:* the Jiang et al. (2019) (J19) concentration-corrected predictor. The x-axis shows the size predicted from the J19 relation $r_{1/2,*} \approx 0.02(c_2/10)^{-0.7} R_{vir}$. The three discrete bands of dots correspond to the three c_2 values in our design, as annotated. Symbol colours indicate halo spin λ . *Right panel:* a simple predictor based on the spin parameter: $r_{1/2,*} \approx 0.5\lambda R_{vir}$. The discrete four narrow bands of dots correspond to the four λ values used in our simulations, as annotated. Symbol colours indicate halo concentration c_2 . For both *Middle* and *Right* panels, the black points indicate the average $r_{1/2,*}$ in each bin, with error bars representing the 1σ scatter. While these simple predictors capture the overall trends, they leave substantial scatter at fixed concentration or spin, indicating that additional parameters are required to explain the full diversity.

ters and the initial baryonic mass fraction regulate galaxy size at fixed halo mass, $M_{vir} = 10^{11} M_\odot$. We have run and analyzed 132 GIZMO FIRE-3 simulations with fixed halo concentration c_2 , inner density slope s_1 , spin λ , and baryon fraction f_b . For each parameter, we studied the 10 stellar half-mass radius, $r_{1/2,*}$, and quantified its dependence on halo properties using complementary approaches, including a quadratic response-surface model and a non-parametric random-forest feature-importance ranking.

Among our diagnostics, halo concentration c_2 emerges as the most informative predictor of galaxy size $r_{1/2,*}$. Halo spin λ provides the second-most contribution, while the inner-slope s_1 exerts a weaker but still significant influence. The baryon fraction f_b plays a distinct role: rather than directly setting the baryonic regulator of size, it primarily modulates how sensitively galaxy size responds to halo structural parameters.

In particular, baryon fractions approaching the cosmic mean lead to centrally concentrated star formation and compact haloes, despite the more extended disc-like morphology implemented in FIRE-3, whereas lower f_b values favor extended, disc-like systems. We further revisited widely used analytic and empirical size predictors by comparing their predictions to our controlled simulations. The Mo et al. (1998) model reproduces the overall scaling of galaxy size remarkably well but systematically underpredicts sizes. This is primarily because the simulated galaxies exhibit low angular-momentum retention factors, $f_j \equiv j_d/m_d \approx 1$, which suggests that galaxy formation preferentially draws on low-angular-momentum baryons and that the initial baryonic mass fraction plays a different role: modulates how sensitively galaxy sizes respond to halo structural parameters.

The empirical concentration-based predictor of Jiang et al. (2019) successfully captures the qualitative trend that galaxies in lower-concentration haloes are larger on average, $r_{1/2,*} \approx 0.02(c_2/10)^{-0.7} R_{vir}$. Our idealized simulations demonstrate that this dependence does not require a cosmological environment, but instead arises naturally from the internal dynamics and galaxy-formation physics within virialized haloes. However, the large scatter in $r_{1/2,*}$ at fixed c_2 underscores the limitations of single-parameter size prescriptions and highlights the need to account for additional halo properties, such as spin, inner structure, and baryon content. Similarly, a simple halo spin-based predictor, $r_{1/2,*} \approx 0.5\lambda R_{vir}$ (Cohen-Williams et al. 2018) also successfully captures the overall trend that galaxies in higher-spin haloes are larger on average, but the large scatter at fixed spin that requires additional modeling.

Our results indicate that the large scatter in $r_{1/2,*}$ at fixed c_2 and λ is not a random noise, but a systematic effect. Including cold gas discs in the galaxy model will systematically increase the sizes, as cold gas discs are more extended, consistent with the expectation that cold gas discs are more extended. Including cold gas discs in the galaxy model will systematically increase the sizes, as cold gas discs are more extended, consistent with the expectation that cold gas discs are more extended. Including cold gas discs in the galaxy model will systematically increase the sizes, as cold gas discs are more extended, consistent with the expectation that cold gas discs are more extended.

The large scatter between these results and single parameter-dependent size relations indicates that all parameters are important contributors to galaxy size. This work establishes a controlled baseline for understanding how secondary halo properties regulate galaxy sizes at the massive-dwarf scale. Future papers in this series will extend this framework to additional halo masses and to other aspects of galaxy morphology, including substructure driven by feedback, such as clumps, cavities, and asymmetries, and will quantify how these features depend on halo structural conditions.

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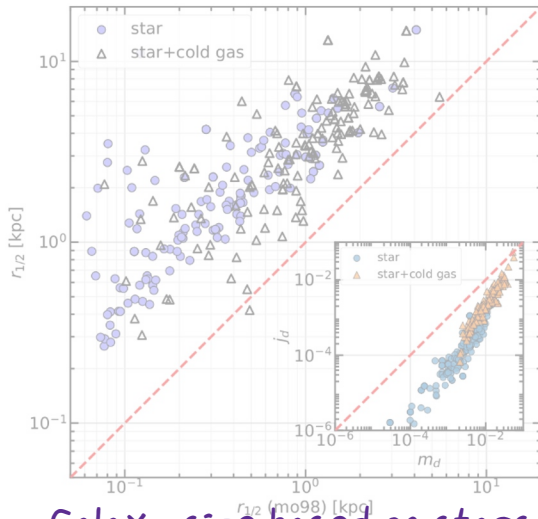


Figure 8. Effect of galaxy definition on the size measurement, and on the comparison with the Mo98 (1998) Mo98 predictions. The vertical axis shows the galaxy size measured in the simulations, while the horizontal axis shows the size predicted by the Mo98 model. Symbols indicate measurements using either stars only (blue circles) or total cold baryons (stars plus cold gas with $T < 10^4$ K; open triangles). The Mo98 predictions are computed with corresponding definition of the disc mass fraction, m_d , and the angular-momentum fraction, j_d . The red dashed line marks equality. *Inset:* angular-momentum fraction j_d versus mass fraction m_d for the two galaxy definitions, with the dashed red line indicating $j_d = m_d$. Including cold gas shifts systems closer to $j_d = m_d$, modestly improving the agreement between the Mo98 predictions and the measured sizes.

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DATA AVAILABILITY

The data products and the analysis pipeline are available upon request.

ALL THE SMART PEOPLE THEY BUILT THEIR IDEAS OFF:

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Want to know the nitty gritty about...

APPENDIX A: QUADRATIC RESPONSE-SURFACE ANALYSIS

A1 Fitting overview

In this Appendix we summarize the technical details of the quadratic response-surface model used in Section 3.2. For each simulation in the $M_{\text{vir}} = 10^{11} M_{\odot}$ grid we consider four halo predictors: concentration c_2 , spin parameter λ , baryon fraction f_b , and inner-slope proxy s_1 . We work with transformed variables that reflect their natural scaling: $\log c_2$, $\log \lambda$, and $\log f_b$ are used, while s_1 is kept on its original (linear) scale. For each transformed predictor u_k we choose a reference value $u_{k,0}$ (the midpoint of the sampled range) and a half-range Δu_k , and define a dimensionless standardized coordinate

$$z_k \equiv \frac{u_k - u_{k,0}}{\Delta u_k}, \quad (\text{A1})$$

so that $z_k \approx -1$ and $+1$ correspond to the low and high settings in the design, and $z_k = 0$ corresponds to the fiducial model. The response variable is the natural logarithm of the 3D stellar halo mass radius, $y \equiv \log r_{1/2,*}$, so multiplicative changes in size become additive shifts in y . In terms of the standardized predictors the quadratic response-surface model can be written as

$$y = \beta_0 + \sum_k \beta_{1,k} z_k + \sum_k \beta_{2,k} z_k^2 + \sum_{i < j} \beta_{i,j} z_i z_j + \varepsilon, \quad (\text{A2})$$

where β_0 is the intercept, $\beta_{1,k}$ are the local linear slopes, $\beta_{2,k}$ describe curvature in each direction, $\beta_{i,j}$ encode pairwise interactions, and ε denotes the residuals. With four predictors this model contains 15 coefficients including the intercept. The chosen coding has the useful property that β_0 represents the mean $\log r_{1/2,*}$ at the midpoints of the design, while the $\beta_{1,k}$ quantify the first-order sensitivity of galaxy size to each halo parameter around this reference point.

We estimate the coefficients by ordinary least squares (OLS), treating the simulation grid as a fixed experimental design. However, we do not rely on the usual OLS formula for uncertainties. Standard residual tests applied to an initial (non-robust) fit reveal significant departures from homoskedasticity: both the Breusch–Pagan and White tests reject the hypothesis of constant residual variance. In this regime we adopt the HC3 heteroskedasticity-consistent covariance estimator as implemented in `statsmodels`. HC3 down-weights high-leverage points more strongly than simpler corrections (HC0/HC1) and yields more conservative small-sample inference. Throughout the paper we therefore quote OLS point estimates for the coefficients, but base all t - and F -tests, confidence intervals, and derived uncertainties on the HC3 covariance matrix. The design is well-conditioned: variance-inflation factors for all regression terms are close to unity ($\text{VIF} \lesssim 1.3$), indicating that collinearity among the standardized predictors is not a concern.

For the full $N = 132$ sample at $M_{\text{vir}} = 10^{11} M_{\odot}$, the quadratic response surface provides a good global description of the size response. The coefficient of determination is $R^2 \approx 0.80$ (adjusted $R^2 \approx 0.78$), with an overall F -statistic $F_{14,117} \approx 36.4$ and $p \approx 5 \times 10^{-86}$. The root-mean-square residual in $\log r_{1/2,*}$ is ≈ 0.37 , corresponding to a 1σ scatter of ≈ 0.16 dex, or a multiplicative factor of ~ 1.5 in physical radius. Only a handful of runs have large studentized residuals or are flagged as influential by Cook’s distance and leverage thresholds. Re-fitting the model after removing these high-influence cases increases R^2 to ≈ 0.86 and leaves the main-effect curves unchanged within uncertainties, indicating that our conclusions are not sensitive to a small subset of outlying realisations.

A2 Importance proxies from coefficient-block tests

Table 2 reports two importance proxies derived from the response-surface model. Both are based on joint Wald tests on blocks of coefficients using the HC3 covariance matrix. For a block test with statistic F , numerator degrees of freedom d , and residual degrees of freedom ν , we convert the result into an effect-size proxy on a 0–1 scale via

$$R^2_{\text{partial}} \equiv \frac{d F}{d F + \nu}. \quad (\text{A3})$$

This quantity is closely related to the “partial η^2 ” used in analysis-of-variance settings; in our context it provides a convenient, approximate measure of the additional (conditional) variance explained by the tested block given the rest of the model. Because it is conditional on the other terms in the regression, it is not additive across factors and should be interpreted comparatively.

For the main-effect proxy we test the joint contribution of the linear and quadratic terms of each factor, $(\beta_{1,k}, \beta_{2,k})$ ($d = 5$). This yields large and highly significant partial R^2 values for c_2 and s_1 (all ≈ 0.46 – 0.49 with $p_F \ll 10^{-6}$), and a much smaller, non-significant value for f_b (partial $R^2 \approx 0.03$ with $p_F \approx 0.14$). For the total-effect proxy we expand the tested block to include the linear and quadratic terms *plus* all interactions involving the factor (five coefficients in total; $d = 5$). In this case, c_2 , λ , and s_1 remain in a high-importance tier (partial $R^2 \approx 0.52$ – 0.58), while the importance of f_b increases substantially (partial $R^2 \approx 0.21$) owing to its coupling with spin. This is consistent with Fig. 6, where the (f_b, λ) panel shows the clearest non-separable behavior.

APPENDIX B: RANDOM-FOREST TRAINING AND FEATURE-IMPORTANCE DEFINITIONS

B1 Model setup and global hyper-parameter tuning

We model the mapping from halo parameters to galaxy size using a random-forest regressor (RF; Breiman 2001). Each RF consists of an ensemble of decision trees trained on bootstrap resamples of the training set, and predictions are obtained by averaging over trees. Because tree-based models are invariant to monotonic feature rescalings and do not require feature standardization, we train directly on the physical input parameters (c_2, s_1, λ, f_b) .

We select the RF hyper-parameters with a single global `GridSearchCV` pass on the full $N = 132$ sample, using 5-fold CV and the negative mean-squared error as the scoring function. The experimental details are

This is details behind their machine learning techniques

```
n_estimators = {300, 600, 800, 1000, 1200, 1500}
max_depth = {None, 5, 10, 15},
min_samples_leaf = {1, 2, 3, 5},
max_features = {sqrt, 1.0}.
```

The best-performing configuration is

```
n_estimators = 1000,
max_depth = 10,
min_samples_leaf = 1,
max_features = sqrt.
```

with bootstrap sampling enabled and `oob_score=True`. These hyper-parameters are then held fixed for all subsequent resampling experiments to avoid confounding the stability of feature importances with repeated re-tuning.

B2 Repeated cross-validation

To assess the stability of the model, we perform repeated cross-validation (CV) on 100 splits (each of size $\sim 10\%$).

On each split, we train a model and evaluate its performance. The mean of $\bar{I}_j$ over the 100 splits is the composite RF importance as normalised estimators (MDI, OOF, SHAP) for each split, followed by the scores show a stable ordering

B3 Feature-importance estimators

We quantify feature importance using four complementary estimators that probe different notions of “importance”:

- (i) **Mean decrease in impurity (MDI).** We use the standard impurity-based importance returned by the RF implementation, which measures the average reduction in within-node variance attributable to each feature, accumulated over all splits and trees.
- (ii) **Out-of-fold (OOF) permutation importance.** For each CV split, we evaluate the baseline validation error (e.g. squared error, MSE_{valid}), then randomly permute a single feature column x_j in the validation set, re-evaluate the error, and re-evaluate MSE_{valid}. The permutation importance is the performance degradation, averaged over repeated shuffles. This estimator directly measures how much predictive accuracy is lost when the information in x_j is destroyed out of sample.
- (iii) **Out-of-bag (OOB) permutation importance.** For each tree in the forest, we identify its OOB subset (samples not included in the bootstrap draw for that tree). We compute a baseline OOB error for that tree, then permute feature x_j within the OOB subset and recompute the error. The per-tree degradation is averaged over trees (and repeated permutations) to obtain an OOB-based permutation importance. Compared to OOF permutation, this avoids using the held-out validation fold and instead exploits the bootstrap structure of the RF.
- (iv) **SHAP-based global importance.** We compute Shapley-style additive explanations (SHAP) for tree ensembles using a TreeExplainer approach (e.g. Lundberg & Lee 2017). For each split we summarise global importance by the mean absolute SHAP value of each feature over the validation fold, $\langle |\text{SHAP}_j| \rangle$, which measures the typical magnitude of the feature’s contribution to the model output.

Because these estimators live on different scales (and permutation scores can occasionally be slightly negative in finite samples), we convert each split-level importance vector into a normalised score that is comparable within a method:

$$\bar{I}_j \equiv \frac{\max(I_j, 0)}{\sum_k \max(I_k, 0)}, \quad (\text{B2})$$

so that $\sum_k \bar{I}_k = 1$ for each estimator and split.

B4 Rank stability, and resulting hierarchy

For the mean of $\bar{I}_j$ over the 100 splits, we compute the composite RF importance as normalised estimators (MDI, OOF, SHAP) for each split, followed by the scores show a stable ordering. f_b is consistently least important. Rank stability: c_2 is ranked first in all splits (depending on estimator, f_b is almost always ranked last. The importance of s_1 varies mildly with the estimator. Metrics tend to favour λ , whereas MDI/SHAP metrics are neutralizing our conservative conclusion that they are in an intermediate tier.

APPENDIX C: TRANSPLANTING THE MO98 FORMALISM TO THE DEKEL–ZHAO HALO PROFILE

C1 A root-finding solver for the Mo98 disc scale length

The classical disc-formation model of Mo et al. (1998, hereafter Mo98) assumes that a fraction m_d of the halo virial mass M_{vir} settles into a rotationally supported exponential disc and retains a fraction j_d of the halo angular momentum.⁵ The disc surface density is

$$\Sigma(R) = \frac{M_d}{2\pi R_d^2} \exp\left(-\frac{R}{R_d}\right), \quad M_d = m_d M_{\text{vir}}, \quad (\text{C1})$$

and the disc angular momentum for a given circular-velocity curve $V_c(R)$ is

$$\begin{aligned} J_d(R_d) &= 2\pi \int_0^\infty \Sigma(R) V_c(R; R_d) R^2 dR \\ &= M_d R_d \int_0^\infty e^{-u} u^2 V_c(u R_d; R_d) du, \end{aligned} \quad (\text{C2})$$

where $u \equiv R/R_d$. The target angular momentum is $J_d = j_d J_h$, with the halo angular momentum J_h from the Mo98 eq. (9),

$$\lambda \equiv \frac{J_h |E_h|^{1/2}}{M_{\text{vir}}} \Rightarrow J_h = \lambda \frac{G M_{\text{vir}}}{|E_h|^{1/2}}, \quad (\text{C3})$$

Writing the halo total energy in the standard form

$$E_h = -\frac{1}{2} f_h M_{\text{vir}} V_{\text{vir}}^2 = -\frac{G M_{\text{vir}}^2}{2 R_{\text{vir}}} f_h, \quad V_{\text{vir}}^2 \equiv \frac{G M_{\text{vir}}}{R_{\text{vir}}}, \quad (\text{C4})$$

one can rewrite eq. (C3) as

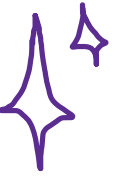
$$J_h = \lambda M_{\text{vir}} R_{\text{vir}} V_{\text{vir}} \sqrt{\frac{2}{f_h}}. \quad (\text{C5})$$

For an NFW halo, Mo98 give $E_h = -(G M_{\text{vir}}^2 / 2 R_{\text{vir}}) f_c$ (Mo98 eq. 22), with

$$f_c(c) = \frac{c}{2} \frac{1 - (1+c)^{-2} - 2 \ln(1+c)/(1+c)}{[c/(1+c) - \ln(1+c)]^2} \quad (\text{C6})$$

(Mo98 eq. 23), where c is the concentration.

⁵ In the main text we often use the retention factor $f_j \equiv j_d/m_d$.



In Mo98 the disc scale length is obtained by rewriting the condition $J_d = j_d J_h$ into a fixed-point form (their eqs. 28–29) and iterating. However, the iterative scheme is not essential: for any specified initial halo profile (NFW in Mo98; DZ in our work), the adiabatic-contraction calculation always starts from the same unperturbed halo, so the mapping $R_d \mapsto J_d(R_d)$ is deterministic. One may therefore solve for R_d directly as a one-dimensional root-finding problem,

$$\mathcal{F}(R_d) \equiv J_d(R_d) - j_d J_h = 0, \quad (\text{C7})$$

where $J_d(R_d)$ is evaluated by direct numerical integration of eq. (C2).

Evaluating $V_c(R; R_d)$ follows Mo98: the halo responds adiabatically to the slow assembly of the disc, conserving $rM(r)$ for each shell,

$$r_i M_i(r_i) = r M_f(r), \quad (\text{C8})$$

where $M_i(r)$ is the initial halo enclosed mass profile, and

$$M_f(r) = M_d(r) + M_h(r), \quad M_h(r) = (1 - m_d) M_i(r_i). \quad (\text{C9})$$

The cumulative mass of an exponential disc is (Mo98 eq. 26)

$$M_d(r) = M_d(R_d) \left[1 - \exp\left(-\frac{r}{R_d}\right) \right]. \quad (\text{C10})$$

The methods they used to match the Mo et al. 1998

$$V_c^2(r) = V_{c,d}^2(r) + V_{c,h}^2(r) = \frac{GM_d(r)}{2R_d} + \frac{GM_h(r)}{r}. \quad (\text{C11})$$

with the thin exponential-disc contribution

$$V_{c,d}^2(r) = \frac{GM_d}{2R_d} y^2 \left[I_0(y) K_0(y) - I_1(y) K_1(y) \right], \quad y \equiv \frac{r}{2R_d}, \quad (\text{C12})$$

where I_n and K_n are modified Bessel functions. In practice, eq. (C7) can be solved robustly with a bracketing method (e.g. Brent's algorithm), and is typically fast on modern hardware while being conceptually more transparent than an explicit fixed-point iteration.

C2 Dekel-Zhao haloes: $M(r)$ and f_{DZ}

To transplant the Mo98 machinery to a Dekel–Zhao (DZ) halo (Freundlich et al. 2020), only two halo-specific ingredients are required: (i) the initial enclosed mass profile $M_i(r)$ entering the adiabatic-contraction step (eq. C8); and (ii) the energy factor f_h entering the spin–angular-momentum conversion (eq. C5).

The DZ density profile is (Freundlich et al. 2020, eq. 11)

$$\rho(r) = \frac{\rho_c}{x^a (1 + x^{1/2})^{2(3.5-a)}}, \quad x \equiv \frac{r}{r_c}, \quad c \equiv \frac{R_{\text{vir}}}{r_c}, \quad (\text{C13})$$

with inner-slope parameter a and concentration c . It is convenient to define the compact variable

$$\chi(x) \equiv \frac{x^{1/2}}{1 + x^{1/2}}, \quad \chi_c \equiv \chi(c) = \frac{c^{1/2}}{1 + c^{1/2}}. \quad (\text{C14})$$

Then the enclosed mass takes the simple closed form

$$M_i(r) = M_{\text{vir}} \left(\frac{\chi}{\chi_c} \right)^{6-2a}. \quad (\text{C15})$$

For the total energy, we define the DZ analogue of the NFW energy factor by

$$E_h \equiv -\frac{GM_{\text{vir}}^2}{2R_{\text{vir}}} f_{\text{DZ}}(a, c) = -\frac{1}{2} f_{\text{DZ}}(a, c) M_{\text{vir}} V_{\text{vir}}^2. \quad (\text{C16})$$

Assuming circular orbits for the virialised halo and using the virial theorem, one obtains

$$f_{\text{DZ}}(a, c) = \frac{2(3-a)c}{\chi_c^{12-4a}} B(\chi_c; 10-4a, 3), \quad (\text{C17})$$

where $B(z; p, q)$ is the (unnormalised) incomplete beta function,

$$B(z; p, q) \equiv \int_0^z t^{p-1} (1-t)^{q-1} dt, \quad (\text{C18})$$

and the integral converges for the parameter range relevant here (in particular $a < 2.5$ so that $p = 10 - 4a > 0$).

With eqs. (C15) and (C17), the Mo98 procedure carries over verbatim: in the contraction step, replace the NFW $M_i(r)$ by the DZ mass profile; in the spin conversion, replace $f_c(c)$ by $f_{\text{DZ}}(a, c)$. The disc scale length R_d is then obtained by solving the same root condition, eq. (C7).

This paper has been typeset from a $\text{\LaTeX}$ file prepared by the author.

(Jayde Willingham aka me!)