



Research Article

Can old sleepy galaxies have huge black holes in them without dark matter?

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Eappen and Kroupa wrote this.

¹In a nutshell:

The formation of supermassive black holes (SMBHs) in early-type galaxies (ETGs) is a key challenge for galaxy formation theories. Using the monolithic collapse models of ETGs formed in Milgromian Dynamics (MOND) from Eappen et al. (2022, MNRAS, 516, 1081, <https://doi.org/10.1093/mnras/stac2249>, arXiv: 2209.00024 [astro-ph.GA]), we investigate the conditions necessary to form SMBHs in MOND and test whether the systems host stars and SMBHs. We study the evolution of the gravitational potential and gas inflow rates in the model relics with a total stellar mass ranging from $0.1 \times 10^{11} M_{\odot}$ to $0.7 \times 10^{11} M_{\odot}$. The gravitational potential and the rapid gas inflow rates in the model galaxies are found to be consistent with the conditions suggested by the model. They also compare it to well-known patterns, like the link between mass and velocity, which come from real observations. The model matches these observations without needing dark matter. It also shows that how gas behaves when a galaxy first forms plays a big role in creating these giant black holes.

Keywords: Galaxy: evolution; galaxy: formation; galaxies: elliptical and lenticular; cD; Galaxies: fundamental parameters; black holes

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¹Background:

Early-type galaxies (ETGs), including ellipticals and lenticulars, are compact spheroid systems characterized by old stellar populations, minimal gas content, and little to no ongoing star formation (Kormendy et al. 2009). These galaxies serve as natural laboratories for understanding galaxy evolution and the co-evolution of supermassive black holes (SMBHs). The scaling relations between SMBHs and galaxy properties, such as the $M_{\text{BH}} - \sigma$ (black hole mass versus central velocity dispersion) and $M_{\text{BH}} - M_{\text{bulge}}$ (black hole mass versus bulge mass) relations, underscore the profound connection between SMBHs and the structural and dynamical properties of ETGs (McConnell & Ma 2013).

SMBHs, with masses exceeding $10^6 M_{\odot}$, are the most massive objects in the universe (Kormendy & Richstone 1995; Kormendy 2001). Observations of high-redshift quasars suggest that these objects formed early in the universe's history, with some SMBHs reaching masses of $10^9 M_{\odot}$ less than a billion years after the Big Bang (Fan et al. 2003; Mortlock et al. 2011; McLure et al. 2013). These findings challenge our understanding of how these massive objects formed and grew. Their best galaxies also formed rapidly, with star formation timescales (SFIS) shorter than 1 Gyr (Thomas et al. 2005; McConnell et al. 2015; Liu et al. 2016; Yan, Jerabkova, & Kroupa 2021). Explaining how stellar-mass black hole seeds grow into SMBHs within these

early-type galaxies remains a key challenge. One option is through interactions in dense stellar environments. Maybe two smaller black holes merge!

These clusters form in the central regions of collapsing gas clouds during galaxy formation, where high gas inflow and high stellar densities drive the central clusters of stellar remnants into a relative regime. This process leads to catastrophic collapse through gravitational wave emission, such a mechanism naturally accounts for the observed correlations between SMBH mass and spheroid mass.

The Milgromian Dynamics (MOND) paradigm offers a compelling alternative for understanding how galaxy formation and evolution. One theory changes how gravity behaves at low accelerations, explaining some observations without dark matter. In some cases, it matches or even outperforms the standard cosmological model. If correct, it could reshape our understanding of galaxy and black hole evolution.

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The biggest black holes are also called supermassive black holes!

the dynamical friction test further questions their existence (Oehm & Kroupa 2024). Within MOND, ETGs are thought to form via the monolithic collapse of primordial pre-galactic gas clouds, producing high stellar densities and rapid star formation (Eappen et al. 2022). Stellar-mass black holes forming in dense central clusters dynamically sink to the centre and coalesce to create SMBHs, as suggested by Kroupa et al. (2020).

Supermassive black hole properties have been linked to the motions at the center of the galaxy.

Simulations of the formation and evolution of ETGs using the RAMSES (PoR) code (Lüghausen, Famaey, & Kroupa 2015; Nageshi et al. 2021) have reproduced many properties of observed galaxies, including the emergence of the downsizing trend in ETGs (Eappen et al. 2022). These simulations provide a platform for studying the formation and evolution of ETGs under MOND and their potential to host SMBHs. However, while the downsizing relations and star formation histories of ETGs in MOND have been explored, the role of the central gravitational potential (Φ_{central}) – a key determinant of SMBH formation – remains unclear.

To test if supermassive black holes could form in early-type galaxies under the MOND model, they check whether the central density is linked to how much gas flows in and how varied the velocities are.

(this is introducing the method!)

In this study, we investigate Φ_{central} and gas inflow rates in ETGs formed in MOND using the PoR code. We aim to determine whether these systems create the necessary conditions for SMBH formation. Our results show that the SMBH masses are consistent with the observed $M_{\text{BH}} \propto \sigma$ scaling relation. To achieve this, we analyse the evolution of the central gravitational potential and gas inflow rates during the early phases of galaxy formation. Additionally, based on the work of Kroupa et al. (2020), we evaluate SMBH masses by assuming that a fraction of the central stellar mass collapses into the SMBH. Finally, we compare the simulated SMBH masses to observed scaling relations to assess whether MOND-modelled ETGs can host SMBHs consistent with empirical data. This work builds on the foundation of Eappen et al. (2022) and aims to connect the dynamics of MOND-modelled ETGs with SMBH formation, providing new insights into galaxy evolution in alternative gravity frameworks.

mass of the stellar particles within a 500 kpc radius. This approach ensures that the centre is dynamically tracked throughout the simulation, independent of any fixed box coordinates.

The simulations are further constrained by a maximum spatial resolution of 0.24 kpc (which is the physical size of an individual grid cell at the highest level of refinement). However, we note that accurately resolving potential gradients typically requires several grid cells. Thus, the effective resolution of our simulation is somewhat larger than the cell size, approximately ≈ 1 kpc in the innermost regions, as determined by the spatial refinement (AMR) capabilities of the PoR code. Improving the spatial resolution to a finer scale would be computationally infeasible due to the significant computational cost associated with higher resolution MOND simulations. While the PoR code in QUMOND mode – the quasi-linear formulation of MOND (Milgrom 2010) in which the modified gravitational field is solved via a two-step Poisson solver indeed requires at most twice the computational time compared to Newtonian physics, the primary limitation for increasing refinement levels in our simulation was the available memory capacity. Higher refinement would substantially increase memory requirements due to the number of cells and particles, and future work will aim to address this with improved computational resources. Despite these limitations, the model allows us to explore key aspects of SMBH formation and galaxy evolution within the framework of MOND. A detailed description of the model formation process, including the initial conditions and simulation setup, can be found in Eappen et al. (2022).

They talk about what simulation sizes are computationally feasible.

Gravitational Potential Equations

The gravitational potential at the centre of the model, Φ_{central} , is calculated in the simulation by solving the MOND-modified Poisson equation. In MOND, the gravitational potential Φ is governed by the equation:

$$\Delta\Phi = 4\pi G \rho_b \left[\frac{|\nabla\Phi|}{a_0} \right]^{-1} \quad (1)$$

where $\Delta\Phi$ is the Laplacian of the gravitational potential, ρ_b is the baryonic matter density, G is the gravitational constant, a_0 is the MOND acceleration constant, ϕ is the Newtonian potential, and $\tilde{v}(y)$ is the MOND transition function, with $y = |\phi|/a_0$. The equation is solved numerically in the QUMOND framework, as implemented in the PoR code. This equation modifies Newtonian gravity at low accelerations ($a < a_0$) by introducing non-linearity in the relationship between mass and potential. The PoR code numerically solves this equation to compute the potential, which is then used to calculate the gravitational pull in a certain area, the first step is to see how much normal matter is packed into that space. This is called baryonic density – “baryonic” just means ordinary stuff like stars, planets, and even people, all made of protons and neutrons.

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From the Newtonian potential, the code computes the phantom dark matter density (ρ_{ph}) that arises due to the MOND modification. This density is given by:

$$\rho_{\text{ph}} = \frac{1}{4\pi G} \nabla^2 \left[a_0 \left(\frac{|\nabla\Phi|}{a_0} \right)^{-1} \right] \quad (2)$$

We also need to know how much phantom matter we would need under a Newtonian physics model.

Methods – How they modelled:

In this study, we use the model galaxies formed with an initial post-Big-Bang gas cloud radius (R_{initial}) of 500 kpc as presented in Eappen et al. (2022). Our analysis focuses on analysing Φ_{central} , gas inflow rates, and estimating the potential mass of a SMBH that could form within the galaxy. For these analyses, we employ the model ‘e38’ which is run at a higher resolution than the comparison to other models. This decision is motivated by the need to capture the critical phases of galaxy formation and evolution during the critical phases of galaxy formation. However, due to computational limitations, we have restricted our analysis to this single high temporal resolution model. For estimating the potential mass reservoir available for SMBH formation, we use the gas mass evolution data from our model (e38), which employs an initial cloud radius ($R_{\text{initial}} = 500$ kpc). Although we mention multiple models in this initial radius (e35, e36, e37, e38 and e39 from Table 1 of Eappen et al. 2022), our SMBH mass estimate is based specifically on the e38 model. The time reported here is relative to the start of the simulation. The simulation box has a side length of 1 Mpc, providing a sufficient volume to follow the collapse and evolution of the isolated galaxy model. Throughout the analysis, we define the ‘central region’ based on the centre of



The total gravitational potential Φ is then recalculated by solving the Poisson equation, which incorporates both ρ_b and ρ_{ph} :

Gravitational potential $\propto$ density of real matter and phantom matter.

$$\Delta\Phi = 4\pi G(\rho_b + \rho_{ph}). \quad (4)$$

Φ_{central} is extracted from the simulation output at specific radii, such as 1, 0.8, and 0.6 kpc, at each time step (The simulation outputs are saved at intervals of 10 Myr, allowing us to follow the evolution of inflow rates and dynamical changes over time. For the period of rapid collapse ($\approx 3\text{--}5$ Gyr), this corresponds to a temporal resolution of 100 snapshots per Gyr). This value represents the depth of the gravitational potential well at the galaxy's centre and evolves as the galaxy forms and stabilises.

Physically, Φ_{central} reflects the gravitational strength in the galaxy's core. A deeper potential well corresponds to a denser and more gravitationally bound region. The evolution of Φ_{central} over time provides insight into the baryonic collapse and the formation of a dense core, processes that are critical for driving gas inflows and forming a SMBH. Additionally, the differences in Φ_{central} at various radii, such as between 1 and 0.6 kpc, reveal the steepness of the potential gradient, which influences the dynamics of gas inflows and central star formation.

By solving the MOND-modified Poisson equation and extracting gravitational potential values at central radii, the simulation provides a detailed measure of Φ_{central} , which is fundamental to understanding galaxy evolution and the conditions for SMBH formation in MOND.

How to calculate how much gas falls to the centre?

To gauge inflows at different radii ($r = 1$ kpc, 0.8 kpc, 0.6 kpc) were computed using data from the simulation. The inflow rate ($\dot{M}_{\text{gas}}$) was calculated as the rate of change of gas mass (M_{gas}) within a given radius over a discrete time interval. For each radius, the inflow rate is given by:

How much gas is falling into the centre can be calculated at each radius and timestep. It is equal to how much mass has been gained on lost over what timeframe at that point in space.

where $\dot{M}_{\text{gas}}(r, t)$ is the inflow rate at radius r and time t , $M_{\text{gas}}(r, t)$ is the gas mass within radius r at time t , and Δt is the time interval between consecutive snapshots. The gas mass is calculated by summing the contributions from individual cells within the specified radius. For each cell, the gas mass is obtained by multiplying the cell volume by its gas density. Cells whose centres lie within the selected radius are included in full. Since the AMR grid structure results in varying cell resolutions, this calculation automatically accounts for local resolution differences. We note that we do not perform partial cell volume calculations for boundary cells; cells are either fully included or excluded based on their centre position.

The simulation data provides $\dot{M}_{\text{gas}}(r, t)$ for discrete time points. Using the differences between consecutive snapshots, the inflow rate is computed for the midpoints of these time intervals to align with the calculated rates.

How to calculate the black hole mass?

theoretically
This analysis estimates the black hole mass (M_{BH}) as a fraction of the central stellar mass and computes the central velocity dispersion (σ) for galaxies within the simulation. The results are compared to the observed $M_{\text{BH}} - \sigma$ scaling relation from McConnell & Ma (2013).

$$r = \sqrt{x^2 + y^2 + z^2}$$

The central stellar mass (M_{central}) is calculated by summing the masses of all stellar particles formed from the gas located within a defined radius ($r_{\text{central}} = 1$ kpc) of the galaxy centre. The distance of each particle from the centre is calculated as:

$$r = \sqrt{x^2 + y^2 + z^2} \quad (6)$$

where x , y , z are the Cartesian coordinates of the particle. Particles with $r \leq r_{\text{central}}$ are selected, and their masses are summed to obtain the central mass:

$$M_{\text{central}} = \sum_{i=1}^N m_i \quad (7)$$

where m_i is the mass of the i -th particle in the central region.

The black hole mass is estimated as a fraction of the central stellar mass. For a given fraction f , the black hole mass is given by:

$$M_{\text{BH}} = f \times M_{\text{central}} \quad (8)$$

Fractions ranging from $f = 0.1$ to $f = 1.0$ are used to explore the potential black hole mass range for the simulated galaxies. These fractions are realistic (Kroupa et al. 2020) as the stellar initial mass function (IMF) depends on the initial conditions and the physical conditions such that the majority of the cluster's mass ends up being in stellar mass black holes (Mack et al. 2022; Joshi et al. 2018; Kroupa et al. 2020; Kroupa et al. 2024).

The 3D central velocity dispersion (σ_{total}) is calculated using the particle velocities in the central region. For each velocity component (v_x , v_y , v_z), the dispersion is computed as:

$$\sigma_x = \sqrt{\frac{1}{N} \sum_{i=1}^N (v_{x,i} - \bar{v}_x)^2}, \quad (9)$$

where N is the number of particles, $v_{x,i}$ is the velocity of the i -th particle, and $\bar{v}_x$ is the mean velocity in the x -direction. Similar expressions are used for σ_y and σ_z . The total 3D central velocity dispersion is:

$$\sigma_{\text{total}} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} \quad (10)$$

The simulated and computed velocity dispersion values are compared to the observed $M_{\text{BH}} - \sigma$ relation from McConnell & Ma (2013).

$$\log_{10}(M_{\text{BH}}) = 8.39 + 1.20 \log_{10}(\sigma / 200), \quad (11)$$

where σ is in km/s. This comparison provides insight into whether the simulated galaxies reproduce the observed scaling relation between black hole mass and central velocity dispersion, thereby validating the model's predictions for SMBH formation.

Results!

Is the gravitational potential well deep enough for stuff to not escape?

We analyse the evolution of the gravitational potential difference at the centre of our model ETG (e38), which forms a total stellar mass of $0.6 \times 10^{11} M_{\odot}$. Fig. 1 shows the evolution of the relative gravitational potential difference relative to the potential

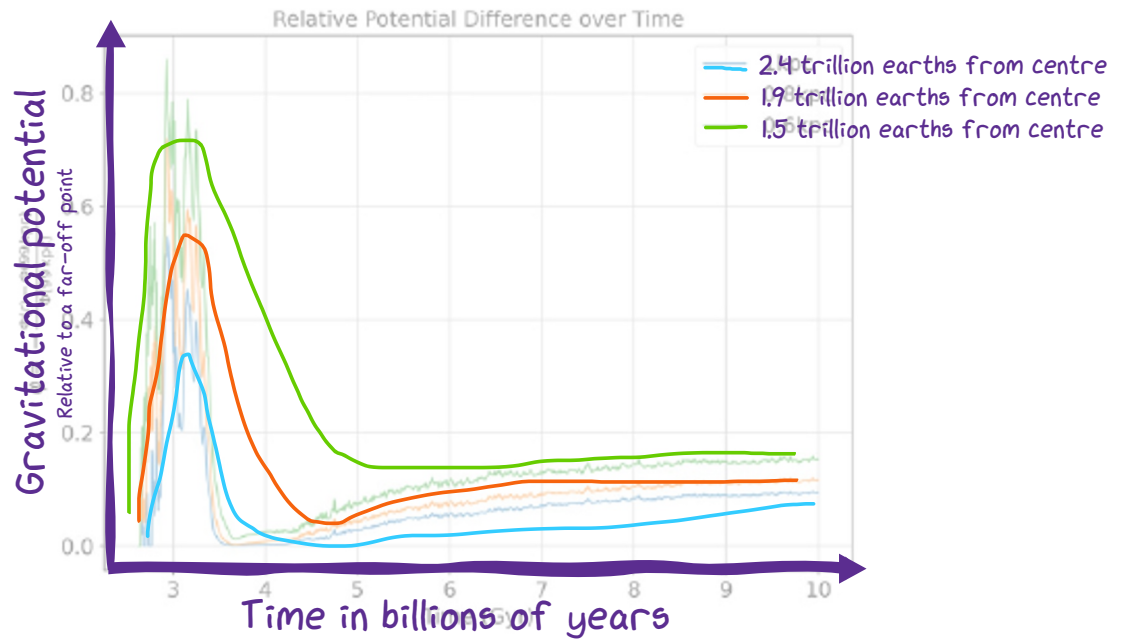


Figure 1. Time evolution of the relative gravitational potential difference between the central region (at radii of 1, 0.8, and 0.6 kpc) and a reference point at 99 kpc. The potential deepens rapidly during the collapse phase (≈ 3 –5 Gyr), stabilising thereafter. The use of potential difference avoids dependence on absolute values and reflects the physical depth of the potential well relative to the galaxy outskirts.

at a radius of 99 kpc between three central radii (1 kpc, 0.8 kpc, and 0.6 kpc) over 10 Gyr.

The reference radius at 99 kpc corresponds to the outskirts of our simulated system, providing a consistent frame of reference for measuring the depth of the central potential well. While an ideal reference would be at infinity, the finite simulation volume necessitates this practical choice. The potential difference highlights the deepening of the gravitational well relative to the galaxy outskirts.

Within the first 5 billion years the gravitational well rapidly gets deeper, allowing mass to fall, particularly when closer to the centre. These are ingredients to create black holes!

Beyond 5 Gyr the potential difference stabilises, indicating that the system transitions to a more dynamically settled state. Notably, the deepest potential difference is seen at smaller radii (0.6 kpc), emphasising the concentration of baryonic mass toward the centre and suggesting that any gas reaching these inner regions could contribute to black hole growth.

To assess the role of MOND dynamics in the evolution of the central potential, we computed the acceleration profile at $t = 4.4$ Gyr, corresponding to the peak of baryonic collapse (Fig. 2). The accelerations within the inner ≈ 2 kpc exceed the MOND acceleration scale a_0 , indicating that the dynamics in the central region are dominated by Newtonian gravity. Notably, the baryonic density, MOND effects become significant at larger radii (≈ 8 kpc and beyond), potentially triggering a final collapse and mass transport toward the centre.

A compelling observational example is the compact ETG NGC 1277 in the Perseus Cluster, with a total stellar mass of $1.2 \times 10^{11} M_\odot$ and an effective radius of approximately 1.2 kpc. This

galaxy hosts a SMBH with a mass of $\approx 1.7 \times 10^{10} M_\odot$, accounting for nearly 14% of its total stellar mass – far above typical scaling relations. Its gravitational potential difference in NGC 1277 is similarly deep, driven by its compact stellar core and massive black hole (Trujillo et al. 2002; van den Bosch et al. 2012; Yıldırım et al. 2017; Ferré-Mateu et al. 2022). This is comparable to the potential difference in our model after ≈ 5 Gyr, supporting the notion that compact ETGs naturally develop deep gravitational wells.

This comparison highlights the potential environment in our simulated system, where the observed properties of real compact ETGs are mirrored. The central potential in both cases underpins the conditions for star formation and growth. The evolution of the potential in our model further supports the idea that compact ETGs can host over-massive SMBHs as a consequence of their early, rapid formation and subsequent stabilisation.

Does enough gas fall into the centre to make a supermassive black hole?

The gas inflow rates at the same radii (1, 0.8, and 0.6 kpc) are illustrated in Fig. 3 for the critical epoch between 4 and 6 Gyr, corresponding to the period of most rapid potential evolution. At a radius of 1 kpc, gas inflow peaks at rates exceeding $2 \times 10^{10} M_\odot/\text{Gyr}$, while smaller radii exhibit similarly high but slightly delayed peaks. These inflows correspond to periods of rapid cooling and contraction of gas toward the centre. The sustained high inflow rate over about half a Gyr suggests that sufficient mass can be funnelled into the inner regions to feed and grow a central SMBH. Notably, around $t \approx 4.7$ Gyr, the central regions experience an outflow phase, as shown in Fig. 3, driven by feedback mechanisms inherent to the model (Eappen et al. 2022), leading to gas heating and temporary disruption of inflows. After 5 Gyr,

Tldr: Yes!

There is also an outflow phase!

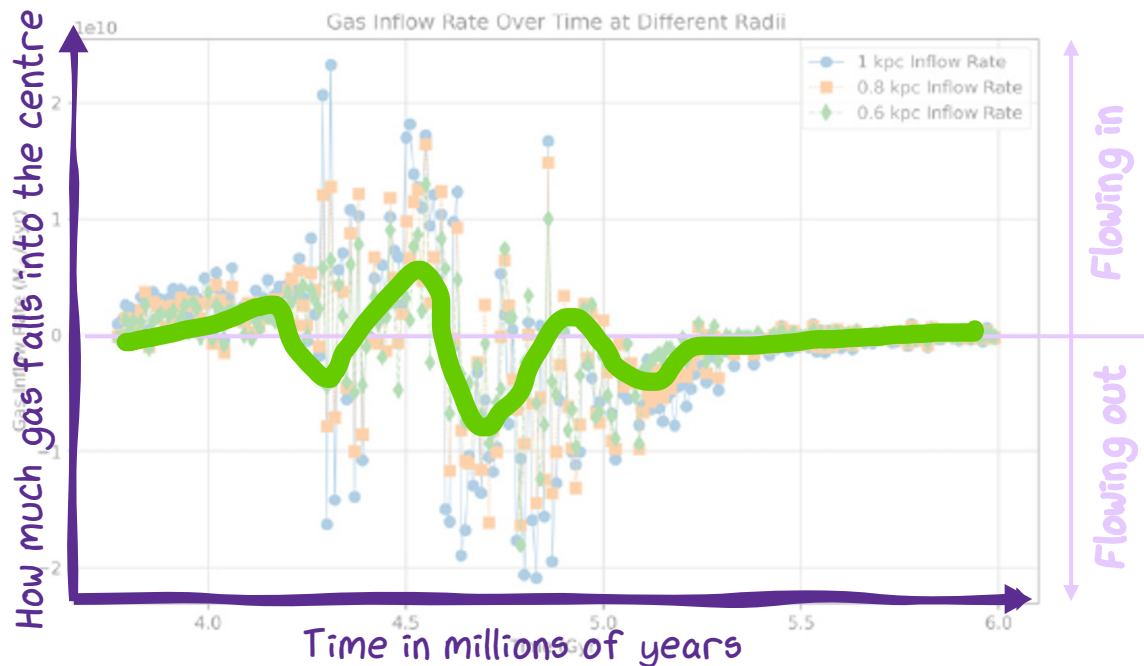


Figure 3. Gas inflow rate over time at different radii (1, 0.8, and 0.6 kpc). The plot illustrates fluctuations in the inflow rate of gas (in $M_{\odot}/\text{Gyr}$) during the time range from 4 to 6 Gyr. Significant variability is observed, particularly in the interval between 4.0 and 5.0 Gyr, with notable peaks and troughs, indicative of dynamic processes affecting gas inflow at these radii.

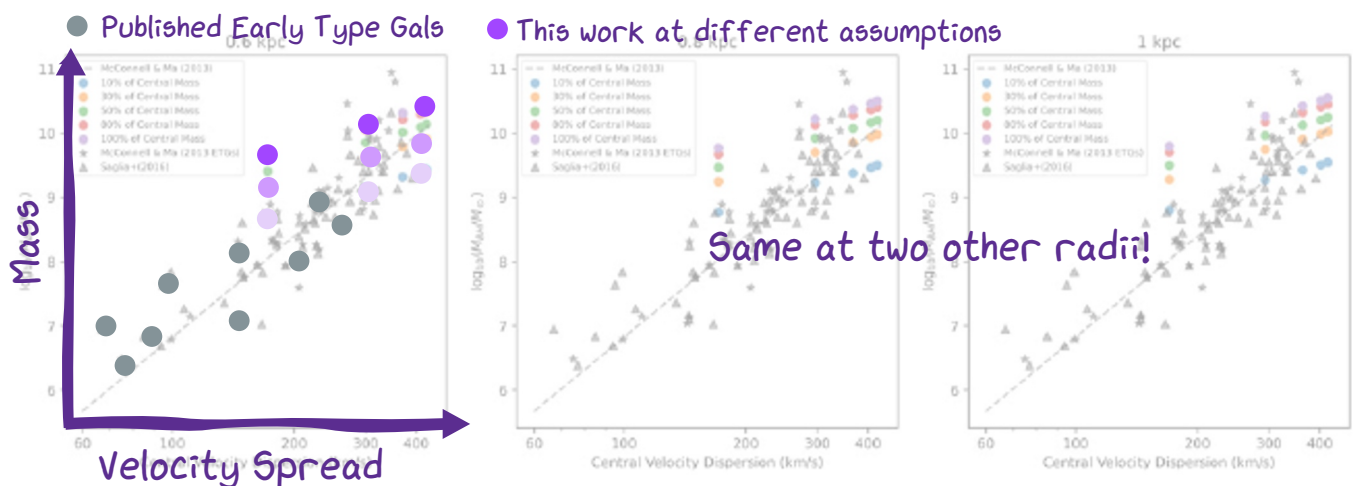


Figure 4. The calculated black hole masses for the model relics (e35, e36, e37, e38 and e39) from Eappen et al. (2022), based on 10%, 30%, 50%, 80%, and 100% of the central stellar mass, are plotted alongside the observed $M_{\text{BH}} - \sigma$ relation from McConnell & Ma (2013). The plotted black stars and black triangles are the ETGs from McConnell & Ma (2013) and Saglia et al. (2016) respectively.

The analysis highlights the capability of MOND dynamics to support the formation of SMBHs in ETGs formed via monolithic collapse. The rapid deepening of the central gravitational potential during the first 0.5 Gyr after begin of the collapse, coupled with high gas inflow rates, creates a conducive environment for black hole seeding and growth.

Quick take away messages:

In this study, we investigated the evolution of ETGs within a MOND framework, focusing on the central gravitational potential, gas inflow rates, and the relationship between black hole mass and central velocity dispersion. Using simulations, we analysed the gravitational and dynamical properties of a model galaxy with a total stellar mass of $0.6 \times 10^{11} M_{\odot}$, providing insights into the

mechanisms governing the formation and growth of SMBHs in compact systems.

The evolution of the central gravitational potential results in a sharp decline during the first 0.5 Gyr after the start of the collapse, corresponding to the rapid depletion of baryonic matter and the formation of the stellar population. This phase is marked by the deepening of the central potential well. The deeper gravitational potential at smaller radii (e.g. 0.6 kpc) underscores the concentration of baryonic matter near the center, creating the conditions necessary for sustained gas inflows and SMBH formation.

Gas inflow rates during the initial epoch (between 4 and 1.5 Gyr after the collapse) exhibit peaks exceeding $2 \times 10^{-2} M_{\odot}/\text{Gyr}$ at 1 kpc, with similarly high but slightly delayed peaks at 0.8 and 0.6 kpc. The subsequent outflow rates and their decline rates after 4.7 Gyr reflects stellar activity and the depletion

Gravitational potential decreases severely very quickly after the collapse of matter.
Deep potential wells close to the centre confirm conditions for gas inflows and properties for the formation of huge black holes.

of gas reservoir, transitioning the system into a quiescent phase. Variations in the adopted feedback prescriptions could significantly influence the inflow dynamics and star formation rates, potentially altering the efficiency of central mass accumulation. Exploring these effects will be the subject of future work. This early period of high inflow rates is crucial for fuelling the growth of the central SMBH and forming a dense stellar core, consistent with the analytical results by Kroupa et al. (2020).

Finally, the relationship between M_{BH} and σ was examined by assuming various fractions (10%, 30%, 50%, 80%, and 100%) of the central stellar mass contribute to the black hole. Black hole masses derived from this align closely with the observed $M_{\text{BH}} - \sigma$ relation, and are in line with MOND model predictions (for $\sigma > 200$ km/s). The results suggest that the combination of a deep gravitational potential, high gas inflow rates, and significant central mass contributions is essential for forming SMBHs consistent with those observed in real galaxies. A more accurate estimate of the SMBH formation potential would require resolving sub-parsec scales and accretion physics.

While our current resolution limits such analysis, the observed collapse and mass assembly provide an upper limit framework for assessing early SMBH formation in MOND.

Overall, our findings demonstrate that ETGs formed within the MOND framework have the potential to naturally reproduce many of the observed properties of ETGs, including their dense cores, quiescent phases, and the $M_{\text{BH}} - \sigma$ relation. These results underscore the importance of early gas inflows and gravitational potential evolution in shaping the growth of SMBHs and the structural properties of ETGs, offering a viable pathway for understanding early structure formation in MOND.

However, we recognise certain limitations in this work. The radii typically around 0.1 kpc or smaller, are not fully resolved in our simulations due to computational limitations. As a result, the work presented here provides only a basic estimate of the conditions for SMBH formation and growth in MOND. This is a simplified approach that does not account for complex processes such as detailed feedback mechanisms, the actual coalescence of the cluster of stellar remnants to a SMBH seed, relativistic effects, or accretion dynamics at smaller scales. Nonetheless, this work offers a foundational framework for future MOND cosmological simulations, highlighting the importance of early gas inflows and gravitational potential evolution in shaping compact ETGs and their central SMBHs. Future efforts to incorporate finer resolutions and additional physics into MOND-based simulations will be critical for achieving a more comprehensive understanding of these processes.

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Data Availability. None.

Where did all these ideas come from?:

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People have been building the puzzle pieces of this paper since 1979!

If you want to know more about certain parts of the method, you will probably find more details here.

Thanks!

Jay

(Jayde Willingham aka me!)